Magnetic Field Instability of a Plasma in a Beam of Electromagnetic Radiation

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A beam of electromagnetic radiation can generate magnetic fields in plasmas. It is shown that those fields grow significantly when the incident radiation is sufficiently strong. We obtain expressions for the characteristic time of the growth of the fields as well as for their spatial distribution and point out a possible mechanism, which can lead to the formation of a quasi-stationary state. The maximum value of the magnetic field strength is estimated

The nonlinear generation of magnetic fields is of importance for the understanding of physical processes in space as well as in laboratory plasmas. Different mechanisms have been proposed as explanations of this effect in various experimental situations (e. g. [1–7]).

The large electro-magnetic fields which occur in laser plasma interactions, are sometimes limited to a cylindrical beam region and are essentially zero elsewhere. The fields are then described by nonlinear waveguide modes if they are strong enough. In the opposite case they may satisfy second order equations (cf. (10)), whose solutions are transformed into the vacuum solutions at the boundary of the beam.

The concentration of the electromagnetic energy into a limited region along the beam axis facilitates the generation of magnetic fields. In the present paper we shall thus consider a process which is caused by the presence of a radially inhomogeneous cylindrical beam of electromagnetic radiation. We observe that fields can be created in a nonuniform plasma (first term on the right hand side of (11)). In addition, however, we demonstrate that magnetic field growth can be important also in a plasma with constant density (second term). The initial phase of the above mentioned process, in which a strong magnetic field appears in the plasma, is then studied, and we discuss a possible saturation mechanism for the instability by taking into account the fact that the growing magnetic fields change the propagation conditions for the external radiation.

Let us thus discuss an electron plasma in the presence of a radially inhomogeneous cylindrical beam, which propagates along the z-axis. We consider only the case where the electric field vector in the plasma initially is mainly in the radial direction of the beam. Then the induced azimuthal quasi-stationary magnetic field B_m generates an additional and significant high-frequency component of the electric field along the z-axis. We also assume that the beam wavelength along the z-axis is much smaller than the characteristic dimension of the beam. As ion motion will be neglected, we use the one-fluid hydrodynamic equations as well as the Maxwell equations. All quantities in these equations are written as sums of rapidly oscillating (no index) and slowly changing parts (index m). A bracket, $\langle\ \rangle$, denotes the average over the period $2\pi/\omega_0$ of the external radiation. The frequency ω_0 is assumed to be larger than the electron Langmuir frequency $\omega_{pe} = (n_0 e^2/\varepsilon_0 m)^{\frac{1}{2}}$. We adopt a complex representation for the rapidly oscillating quantities so that, for example, the electric field $\boldsymbol{E}(\boldsymbol{r}, t)$ is expressed in terms of the complex amplitude $\boldsymbol{E}(r)$ as $\boldsymbol{E}(r, t) = \boldsymbol{E}(r)\exp(i\,\omega_0 t - i\,k_0 z) +$ compl. conj. The slowly changing variables are however essentially real. Our basic system of equations is then

$$i\,\omega_0 \boldsymbol{v} \approx \frac{e}{m}\,(\boldsymbol{E} + \boldsymbol{v}\,x\,\boldsymbol{B}_m) - \nu\,\boldsymbol{v}\,, \tag{1}$$

$$\nabla\,x\,\boldsymbol{E} = -i\,\omega_0 \boldsymbol{B}\,, \tag{2}$$

$$\nabla\,x\,\boldsymbol{B} = \frac{i\,\omega_0}{c^2}\boldsymbol{E} + \mu_0 e\left[n_0 \boldsymbol{v} + \frac{i}{\omega_0}\boldsymbol{v}_m \nabla\cdot(n_0 \boldsymbol{v})\right] \tag{3}$$

$$\frac{\partial \boldsymbol{v}_m}{\partial t} + v_{mr}\frac{\partial}{\partial r}\boldsymbol{v}_m + \langle \boldsymbol{v}\cdot\nabla\,\boldsymbol{v}^* + \boldsymbol{v}^*\cdot\nabla\,\boldsymbol{v}\rangle \tag{4}$$

$$= \frac{e}{m}\,(\boldsymbol{E}_m + \boldsymbol{v}_m\,x\,\boldsymbol{B}_m + \langle \boldsymbol{v}\,x\,\boldsymbol{B}^* + \boldsymbol{v}^*\,x\,\boldsymbol{B}\rangle) - \nu\,\boldsymbol{v}_m$$

$$\partial E_{mz}/\partial r = \partial B_m/\partial t\,, \tag{5}$$

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$$\nabla x \boldsymbol{B}_m = \frac{1}{c^2}\frac{\partial \boldsymbol{E}_m}{\partial t} + \mu_0 e \tag{6}$$
$$\cdot \left[n_0 \boldsymbol{v}_m + \frac{i}{\omega_0} \langle \boldsymbol{v}^* \nabla \cdot (n_0 \boldsymbol{v}) - \boldsymbol{v} \nabla \cdot (n_0 \boldsymbol{v}^*) \rangle \right],$$

where ν, $\boldsymbol{v}$, $\boldsymbol{E}$ and $\boldsymbol{B}$ denote the electron-ion collision frequency, velocity and electric and magnetic fields, respectively. From (1) – (3) one then finds the following expressions for the complex amplitude of the longitudinal component of the electric field E_z, as well as for the velocity components v_r and v_z, in terms of the amplitude E_r:

$$E_z = \frac{i\,\Omega}{\omega_0}\left(1 - \frac{\omega_0^2 - \Omega^2}{\delta^2 \Delta}\right) E_r - \frac{i\,c^2 k_0 (\omega_0^2 - \Omega^2)}{\omega_0^2 \delta^2} \frac{1}{r}\frac{\partial}{\partial r} r\, E_r, \tag{7}$$

$$v_r = - \frac{i\,e}{m\,\omega_0}\left(1 + \frac{\Omega^2}{\delta^2 \Delta}\right) E_r - \frac{i\,e\,c^2 k_0\, \Omega}{m\,\omega_0^2 \delta^2} \cdot \frac{1}{r}\frac{\partial}{\partial r} r\, E_r + \frac{i\,e\,c^2\,\Omega^2\,\omega_{\rm pe}^2}{m\,\omega_0^3\,\delta^4}\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial E_r}{\partial r}, \tag{8}$$

$$v_z = - \frac{i\,\omega_0}{\Omega} v_r + \frac{e}{m\,\Omega} E_r, \tag{9}$$

where $\Omega(r,t) = e\,B_m/m$ is the electron cyclotron frequency,

$$\delta^2 = \omega_0^2 - \omega_{\rm pe}^2 - \Omega^2 - i\,\nu\,\omega_0 (2 - \omega_{\rm pe}^2/\omega_0^2),$$
$$\Delta = 1 - \omega_{\rm pe}^2 k_0\, v_{mz}/\omega_0\,\delta^2$$

and E_r satisfies the equation

$$\frac{\partial^2 E_r}{\partial r^2} + \frac{1}{r}\frac{\partial E_r}{\partial r} - \left\{ \frac{|\delta|^2}{k_0^2 c^2}\left[k_0^2 - \frac{\omega_0^2}{c^2} + \frac{\omega_{\rm pe}^2}{c^2}\left(1 + \frac{\Omega^2}{|\delta|^2}\right)\right] + \frac{\omega_{\rm pe}^2\,\Omega}{\omega_0\,c^2 k_0\, r} + \frac{1}{r^2}\right\} E_r = 0. \tag{10}$$

Inserting (7) – (9) in (4) and (6) and solving the resulting system with respect to $\boldsymbol{E}_m$, it is now possible, by means of (5), to find the equation which describes the spatial distribution and the time-evolution of the azimuthal magnetic field strength B_m. Neglecting small terms ($\partial \boldsymbol{v}_m/\partial t$ in (4)) and assuming v_{mz} to be small (i.e. Δ is close to unity) we then obtain

$$\frac{\partial \Omega}{\partial t} = c^2 \frac{\partial}{\partial r}\left[W \frac{k_0}{\omega_0^3}\frac{\partial \omega_{\rm pe}^2}{\partial r} + W \frac{k_0}{\omega_0\,\Omega}\frac{\partial \Omega}{\partial r} + \frac{\nu}{\omega_{\rm pe}^2}\frac{1}{r}\frac{\partial}{\partial r} r\,\Omega\right], \tag{11}$$

where

$$W = \frac{i\,e^2}{m^2\,\omega_0\,|\delta|^2}\left\langle E_r \frac{\partial E_r^*}{\partial r} - E_r^* \frac{\partial E_r}{\partial r}\right\rangle.$$

We can see from the first term on the right hand side of (11) that, in the presence of a density gradient, it is possible to generate magnetic fields due to gradients in the electromagnetic field. We shall here focus our interest on a uniform plasma, however, and the first term is thus zero. Below, we then solve (11) in the region where the growth of the magnetic field (second term) is more important than the diffusion along the beam radius (third term). That limitation corresponds to a threshold for the present effect. We denote $\Omega(r = r_0, t = 0)$ by Ω_0. A simple solution of (11) is now

$$\Omega(r,t) \approx (\Omega_0 + \omega_0\, t/t_0)\,\frac{g(r_0)}{g(r)} \exp(r_0 - r)/R, \tag{12}$$

where

$$g(r) = (1 + \exp(-r/R))^2,$$
$$t_0 = - \frac{\omega_0^2 R^2 g(r_0)}{2\,k_0\,c^2\,W} \exp r_0/R.$$

and

$$R = r_0 \left[\ln \frac{1 - R\left[\frac{\partial}{\partial r}\frac{\Omega(r)}{\Omega_0}\right]_{r_0,\,t=0}}{1 + R\left[\frac{\partial}{\partial r}\frac{\Omega(r)}{\Omega_0}\right]_{r_0,\,t=0}} \right]^{-1}.$$

From (12) we note that the time scale for the magnetic field growth decreases when the electromagnetic field gradients increase. Expression (12) is however only valid for our particular, although rather realistic, choice of $\Omega(r, t = 0)$, and the result is of course more complicated if the initial magnetic field is arbitrary.

A somewhat similar effect has already been found in plane geometry by Abdullaev et al. [7], but their characteristic time of growth of the magnetic field turns out to be proportional to $(\nu W)^{-1}$. In our derivation, where it is essential to keep previously neglected terms in (4), the growth time (see [12]) is independent of $\nu (t_0 \sim W^{-1})$. Our result is thus significantly different from that of [7].

The change in the propagation conditions for the electromagnetic waves, which is caused by the growing magnetic field, may lead to a stable quasi-stationary state. It can be seen from (11) that it is necessary to include magnetic field effects in δ when the electron cyclotron frequency Ω is comparable to $(\omega_0{}^2 - \omega_{pe}^2)^{1/2}$, which means that the solution of (11) will be a complicated function of Ω. It is then necessary to use that modified solution in the equation for E_r, observing that propagation of radiation is impossible when $\Omega^2 > \omega_0{}^2 - \omega_{pe}^2$. In order to discuss the stationary state we thus have to find the self-consistent solution of (10) – (12), with proper boundary conditions. A very crude estimate of the corresponding magnetic field can however be obtained by means of the arguments above concerning the change in the solutions with the growth of B_m. Thus, when $\omega_0 - \omega_{pe} \gg \nu$, one finds that $\Omega^2(t = \infty) \approx \omega_0{}^2 - \omega_{pe}^2$.

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